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ON THE DRAG FORCE OF A SOLID SPHERE IN POWER LAW FLOW MODEL

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Abstract-A theoretical approach for estimation of the drag correlation coefficients in the flow of Newtonian or weak non-Newtonian liquids around spherical solid particles is presented. Some new analytical relations are derived by using a modified stream function. It is shown that these relationships can be applicable in a wide range of Reynolds number, up to $Re < 1000$ for the liquids with a flow behaviour index, n, in the range of $0.5 < n < 1$. The predicted coefficients from these relationships are in very good agreement with the experimental data given in the literature.

1. INTRODUCTION

Fluid flow around the submerged objects is encountered in various kinds of engineering applications such as mechanical or magnetic separations, and in flow through beds of solids. In such systems, it is generally immaterial which phase, solid or fluid is assumed to be at rest, and it is the relative velocity between the two that is important. The literature data indicated that the effects of the fluid flow behavior index, n (where $n = 1$ for Newtonian liquids) on the drag coefficients, C_D, is considerable especially in the range of $Re < 1$. Since generally $Re < 1$ in many engineering applications, such as sedimentation, magnetic filtration etc., the determination of $C_D = f(n, Re_n)$ relationship in this range of Re is of importance [1-3].

It is known that various solutions and mixtures are non-Newtonian, and the shear stress is not directly proportional to the velocity gradient. One of the most commonly used models to describe the non-Newtonian behavior is represented by the power law equation (Ostwald-de Waele equation). It may be derived from the equation of motion written for non-Newtonian liquids that C_D is a function of both Re_n and n, [4-11].

$$C_D = 24\, X_n / Re_n \tag{1}$$

where X_n is a drag correction factor as a function of n such that $X_n(n) = X_n$ and $X_n(1) = 1$. Re_n is the modified Reynolds number defined as

$$Re_n = \rho_f\, d^n\, V_\infty^{(2-n)} / \kappa \tag{2}$$

where κ is fluid consistency index.

Due to insufficient experimental data, there is no general agreement about the terms of the X_n function. Many workers tried to solve the equation of motion using different flow models for non-Newtonian liquids to derive an expression for X_n. Based on these solutions or experimental data, various functions for X_n have been proposed in the literature for the flow conditions of $Re_n \ll 1$. For example, Kawase and Ulbrecth [8] proposed for $|n-1| \ll 1$

$$X_n = 3^{1.5(n-1)} \frac{2 + 29n - 22n^2}{n(2n+1)(n+2)} \tag{3}$$

Leonov and Isaev [10] proposed

$$X_n = 3^{n-1} \tag{4}$$

Moshev [11] proposed

$$X_n = (0.832)^{\frac{n-1}{2}} \tag{5}$$

It may be seen that the values of X_n estimated from above equations are inconsistent. For example, Kawase and Ulbrecth [8] model predicts always $Xn > 1$, Leonov and Isaev [10] model predicts always $Xn < 1$, but Moshev [11] model predicts always $Xn \approx 1$ for all values of $n < 1$. On the other hand, theoretical calculations and some experimental data indicated that especially for the liquids of $n < 0.7$, the X_n may assume values as $X_n < 1$ [7, 9]. For example, Lali et al. [9] investigated the motion of a spherical particle in the solutions of CMC (carboxy methylcellulose) in a relatively wide range of Re_n. Their results indicate that $X_n > 1$ in the range of $Re_n \approx 1$, but $X_n < 1$ in the range of $Re_n << 1$. Several other studies indicate that the X_n may assume values such that $X_n > 1$, or $X_n < 1$, or $X_n \approx 1$ depending on n and Re_n. However, non of the functions given above satisfies all the three possibilities. Therefore, a development of an analytical relation giving a general $C_D = f(n, Re_n)$ relationship which may assumes above mentioned three values will be useful in many practical applications involving flow of non-Newtonian liquids over solid bodies.

Several approximation formulas are proposed in literature to adopt the correlation coefficient for higher range of Re_n, and to take into account the dependence of X_n on Re_n. Some of them are rather complicated, but instead of Eqs.(3)-(5), the following equation is being widely used for the range $10^{-3} \leq Re_n \leq 10^3$ [8]

$$C_D = \frac{24X_n}{Re_n} + \frac{f_2(n)}{Re_n^{f_3(n)}} \tag{6}$$

where X_n is the drag correlation coefficient from one of the models given above(Eqs.(3)-(5)), $f_2(n)$ and $f_3(n)$ are functions of n such that $f_2(n) = 10.5\,n - 3.5$ and $f_3(n) = 0.32\,n + 0.13$. However, the validity of this equation has not been established due to lack of experimental data. A survey of literature data indicated that this equation is also not sufficiently good to approximate experimental data for a relatively high range of Re_n.

The aim of this paper is to develop some new $C_D = f(n, Re_n)$ relationships providing a better approximation to the experimental data. For this derivation the equation of motion has been solved by integral method using a flow model for non-Newtonian liquids.

2. FORMULATION OF THE PROBLEM FOR THE DRAG COEFFICIENT FOR SPHERICAL SOLID PARTICLES

To determine the drag force (F_D) and its the lift (normal) and drag (tangential) components acting on a particle, the velocity profiles and pressure drops around the particle must be known. The components of F_D for Newtonian liquids may be determined by using the equation of motion or the Navier-Stokes equations. This equation must be modified to determine the components of F_D in the flow of non-Newtonian liquids. In the studies of flow of non-Newtonian liquids around spheres, generally following assumptions were made:

1. The liquid density and consistency index are constant.

2. The flow over the sphere is characterized by creeping flow, i.e., $Re_n << 1$.
3. The liquid has weak non-Newtonian properties and its rheological properties may be represented by the power law equation as in Ostwald-de Waele model, provided that $|n - 1| << 1$.
4. The effective body forces is gravitational force ($F_G = g$, where g is gravity of acceleration). All the other forces (e.g., adhesion, electrical etc.) and diffusion are neglected.
5. The velocity profiles of the non-Newtonian liquid around the sphere, in the first approximation, is identical with that of a Newtonian liquid.

Basically, the first 4 assumptions are also valid in this paper. However, instead of assumption 5, it is assumed in this paper that a mathematical description of the stream function (Ψ) of a non-Newtonian liquid past a sphere may be represented by [12]

$$\Psi = \frac{V_r}{2} r^2 \left[1 - \frac{3}{2}\left(\frac{a}{r}\right)^n + \frac{1}{2}\left(\frac{a}{r}\right)^{3n} \right] \mathrm{Sin}^2\theta \qquad (7)$$

where a is the radius of spherical particle, r and θ are spherical coordinates from the center of particle.

Based on Eq.(7), the radial and tangential velocity components (V_r, V_θ) are determined as

$$\ldots \frac{\partial\Psi}{\ldots} \ldots \left[\ldots \frac{3}{\ldots}\left(\frac{a}{\ldots}\right)^n \ldots \frac{1}{\ldots}\left(\frac{a}{\ldots}\right)^{3n} \right] \mathrm{Co}\ldots$$

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$$\tau = \ldots \tau' = -p\bar{I} + 2\left[\ldots \left|(2I_2^*)^{\frac{1}{2}}\right|^{n-1} \right] \ldots \qquad (\ldots)$$

where τ' is additional stress tensor for non-Newtonians, p is static pressure, $\vec{I}$ is the unit tensor, I_2^* is modified second invariant for non-Newtonians. I_2^* is dependent on the rate deformation tensor (Δ_{ij}) as:

$$I_2^* = \sum_{i;j} \Delta_{ij}^2, \tag{11a}$$

where i, j, refers to x, y, z, in Cartesian coordinates, or refers to r, θ, φ in spherical coordinates. The rate of deformation tensor is defined as

$$\Delta_{ij} = 0.5\left[\left(\frac{\partial V_i}{\partial X_j}\right) + \left(\frac{\partial V_j}{\partial X_i}\right)\right] \tag{11b}$$

Inserting Eqs. (10) and (11) and the assumption of F = g into Eq.(9) gives

$$\frac{d\vec{v}}{dt} = -\frac{1}{\rho_f}\nabla P + \frac{2}{\rho_f}\nabla\left[\left(\kappa\left|\left(2I_2^*\right)^{\frac{1}{2}}\right|^{n-1}\right)\vec{\Delta}\right] + \vec{g} \tag{12}$$

Since we have assumed $|n - 1| \ll 1$ and $Re_n \ll 1$, the modified second invariant, I_2^*, may be approximated as:

$$I_2^* = \Delta_{rr}^2 + \Delta_{\theta\theta}^2 + \Delta_{\varphi\varphi}^2 + 2\Delta_{r\theta}^2 \approx \frac{27V_\alpha^2}{8r^2}\left(\frac{a}{r}\right)^{\frac{2}{n}} \cos^2\theta \tag{13}$$

Let us suppose, the origin of the spherical coordinate system, (r, θ, φ) is the center of the spherical particle and the polar axes is in the direction fluid flow. In addition, let us define a quantity such that $P = p + \rho_f \vec{g}\, \vec{r}$. Here P represents the combined effect of static pressure and gravitational force. By taking into account the conditions of symmetry, ($\partial/\partial\varphi = 0$, $V_\varphi = 0$), Eq. (12) may be rewritten in term of τ' with its r and θ components [1]

r-component

$$\frac{\partial P}{\partial r} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\tau'_{rr}\right) + \frac{1}{r\mathrm{Sin}\theta}\frac{\partial}{\partial\theta}\left(\tau'_{r\theta}\mathrm{Sin}\theta\right) - \frac{\tau'_{\theta\theta} + \tau'_{\varphi\varphi}}{r} \tag{14a}$$

θ-component

$$\frac{1}{r}\frac{\partial P}{\partial\theta} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\tau'_{r\theta}\right) + \frac{1}{r\mathrm{Sin}\theta}\frac{\partial}{\partial\theta}\left(\tau'_{\theta\theta}\mathrm{Sin}\theta\right) + \frac{\tau'_{r\theta} - \tau'_{\varphi\varphi}\cot\theta}{r} \tag{14b}$$

Inserting Eq. (13), and Eq. (10) into Eq. (14) and after some mathematical rearrangements the function P can easily be determined since dP is also stated as

$$dP = \frac{\partial P}{\partial r}dr + \frac{\partial P}{\partial\theta}d\theta. \tag{15}$$

Substitution of Eq. (15) into this equation and then integration with the boundary conditions that P = P at r = r, and $P = P_\infty$, at $r = \infty$, gives the pressure distribution around the sphere. This resultant relation for the pressure on the surface of sphere (r = a) is then where τ' is additional stress tensor for non-Newtonians, p is static pressure, $\vec{I}$ is the unit tensor, I_2^* is modified second invariant for non-Newtonians. I_2^* is dependent on the rate deformation tensor (Δ_{ij}) as:

$$I_2^* = \sum_{i;j} \Delta_{ij}^2 , \tag{11a}$$

where i, j, refers to x, y, z, in Cartesian coordinates, or refers to r, θ, φ in spherical coordinates. The rate of deformation tensor is defined as

$$\Delta_{ij} = 0.5\left[\left(\frac{\partial V_i}{\partial X_j}\right)+\left(\frac{\partial V_j}{\partial X_i}\right)\right] \tag{11b}$$

Inserting Eqs. (10) and (11) and the assumption of F = g into Eq.(9) gives

$$\frac{d\vec{v}}{dt} = -\frac{1}{\rho_f}\nabla P + \frac{2}{\rho_f}\nabla\left[\left(\kappa\left|(2I_2^*)^{\frac{1}{2}}\right|^{n-1}\right)\vec{\Delta}\right] + \vec{g} \tag{12}$$

Since we have assumed $|n - 1| << 1$ and $Re_n << 1$, the modified second invariant, I_2^*, may be approximated as:

$$I_2^* = \Delta_{rr}^2 + \Delta_{\theta\theta}^2 + \Delta_{\varphi\varphi}^2 + 2\Delta_{r\theta}^2 \approx \frac{27V_\alpha^2}{8r^2}\left(\frac{a}{r}\right)^{\frac{2}{n}}\cos^2\theta \tag{13}$$

Let us suppose, the origin of the spherical coordinate system, (r, θ, φ) is the center of the spherical particle and the polar axes is in the direction fluid flow. In addition, let us define a quantity such that $P = p + \rho_f \vec{g}\ \vec{r}$. Here P represents the combined effect of static pressure and gravitational force. By taking into account the conditions of symmetry, ($\partial/\partial\varphi = 0$, $V_\varphi = 0$), Eq. (12) may be rewritten in term of τ' with its r and θ components [1]

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$$\frac{\partial P}{\partial r} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\tau'_{rr}\right) + \frac{1}{r\mathrm{Sin}\theta}\frac{\partial}{\partial\theta}\left(\tau'_{r\theta}\mathrm{Sin}\theta\right) - \frac{\tau'_{\theta\theta}+\tau'_{\varphi\varphi}}{r} \tag{14a}$$

θ-component

$$\frac{1}{r}\frac{\partial P}{\partial\theta} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\tau'_{r\theta}\right) + \frac{1}{r\mathrm{Sin}\theta}\frac{\partial}{\partial\theta}\left(\tau'_{\theta\theta}\mathrm{Sin}\theta\right) + \frac{\tau'_{r\theta} - \tau'_{\varphi\varphi}\cot\theta}{r} \tag{14b}$$

Inserting Eq. (13), and Eq. (10) into Eq. (14) and after some mathematical rearrangements the function P can easily be determined since dP is also stated as

$$dP = \frac{\partial P}{\partial r}dr + \frac{\partial P}{\partial\theta}d\theta . \tag{15}$$

Substitution of Eq. (15) into this equation and then integration with the boundary conditions that P = P at r = r, and $P = P_\infty$, at $r = \infty$, gives the pressure distribution around the sphere. This resultant relation for the pressure on the surface of sphere (r = a) is then

$$P - P_{\infty} = -\frac{3\kappa}{4}\left(\frac{27}{4}\right)^{\frac{n-1}{2}}\left(\frac{V_{\infty}}{a}\right)^{n}\left(n^{2} - 5n + 5\right)n\cos\theta\left|\cos\theta\right|^{n-1} \tag{16}$$

Equation (17) is the basic equation to develop a relationship giving the drag force exerted by a non-Newtonian liquid on a spherical particle. The magnitude of the total drag force is computed by integrating the following equation after the components of τ_{rr} and $\tau_{r\theta}$ are inserted into it. The result is the total force as:

$$F = \int_{0}^{2\pi}\int_{0}^{\pi}\left(\tau_{rr}\text{Cos}\theta - \tau'_{r\theta}\text{Sin}\theta\right)\Big|_{r=a} a^{2}\text{Sin}\theta d\theta d\varphi \,. \tag{17}$$

After some mathematical rearrangements, the resultant force over the surface of sphere is obtained as

$$F = 6\pi a^{2}\kappa\left(\frac{27}{4}\right)^{\frac{n-1}{2}}\left(\frac{V_{\infty}}{a}\right)^{n}\frac{\left(n^{2} - 5n + 7\right)}{n+2} \tag{18}$$

Due to the symmetry conditions, the direction of this force is the same with that of the fluid flow at far from the surface (infinity). The drag coefficient is defined by means of Eq. (18) as

$$C_{D} = \frac{F}{\frac{1}{2}\rho_{f}V_{\infty}^{2}\pi a^{2}} = \frac{24}{Re_{n}}X_{n}, \tag{19a}$$

$$X_{n} = 3^{1.5(n-1)}\frac{\left(n^{2} - 5n + 7\right)}{n+2} \tag{19b}$$

4. RESULTS AND DISCUSSION

Most of the approximation formulas given in the literature are restricted, in application, to a narrow range of Re_n [6,7,10,11]. Equation (19) shows a better approximation to experimental data in the range of $Re_n < 1$. In order to apply Eq. (19) for the higher range of Re_n we propose the following equations instead of Eq.(6)

$$C_D = 24\ X_n^{*}/Re_n \tag{20a}$$

where X_n^{*} is a modified correlation coefficient such that

$$X_n^{*} = X_n \qquad \text{for } Re_n < 1\text{x}10^{-5} \tag{20b}$$

$$X_n^{*} = X_n + \frac{1-n}{2n+1}\log\left(10^{5}\,Re_n\right) \qquad \text{for } 1\text{x}10^{-5} \le Re_n < 1\text{x}10^{-3} \tag{20c}$$

$$X_n^{*} = X_n + \frac{f(n)}{24\,Re_n^{g(n)}} \qquad \text{for } 1\text{x}10^{-3} \le Re_n < 1\text{x}10^{3} \tag{20d}$$

where X_n is from Eq (19), $f(n) = 4n^2$ and $g(n) = (n-3)/3$

A comparison of the predicted coefficients from this equation with the experimentally determined coefficient is summarized in Table 1.

Table 1 A comparison of the experimentally determined X_n values with the model predictions.

Experimental Data				X_n values predicted from the models			
n	Re_n	C_D	X_n	Kawase and Ulbrecht [8]	Leonov and Isaev [10]	Moshev [11]	Eq. (20)
0.56	1.35×10^{-5}	8.50×10^{5}	0.478	1.807	0.617	1.041	0.466
0.56	2.46×10^{-5}	5.16×10^{5}	0.528	1.807	0.617	1.041	0.520
0.56	6.40×10^{-5}	2.20×10^{5}	0.587	1.807	0.617	1.041	0.606
0.56	3.57×10^{-4}	4.27×10^{4}	0.635	1.807	0.617	1.041	0.761
0.56	1.48×10^{-3}	1.47×10^{4}	0.907	1.807	0.617	1.041	0.889
0.56	0.024	1.03×10^{3}	1.000	1.815	0.625	1.049	0.920
0.74	0.61	39.5	1.004	1.609	0.882	1.154	0.978
0.696	1.48	17.3	1.067	1.761	0.921	1.233	1.009
0.82	6.14	5.6	1.433	1.980	1.462	1.658	1.360
0.89	22.8	2.2	2.090	2.729	2.404	2.528	2.172

Experimental data are from Lali et al. [9]

Table 1 suggest that Eq.(20) provides a better approximation to the experimental data than the models given in the literature. A plot of Eq. (20) for $n = 0.56$ and $n = 0.84$, and the standard drag curve of Newtonian liquids are represented in Figure 1 for the range of $10^{-3} < Re_n < 10^3$.

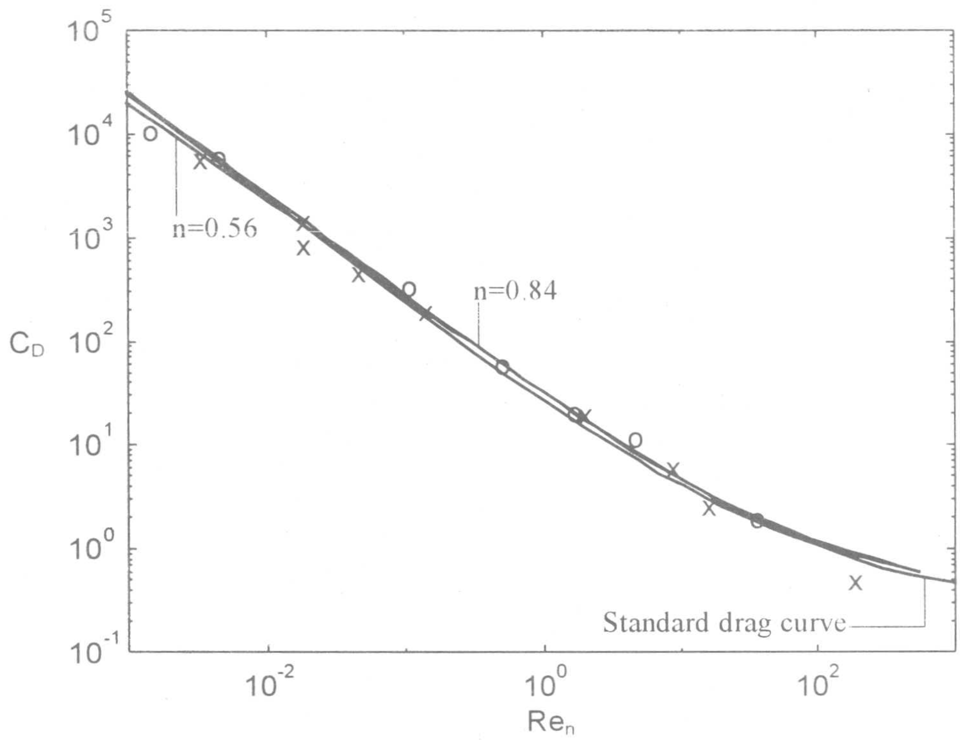

Figure 1. Representation of $C_D = f(n, Re_n)$ relation from Equation (20) of the present work and the standard Newtonian drag curve. The experimental data from Lali et al. [9].

The figure indicates that the X_n values predicted from Eq. (20) assume all the three possible values of $X_n > 1$, $X_n = 1$ or $X_n < 1$, depending on Re_n Similar plots can be obtained for other values of n. The experimental data from Lali et al. [9] are also presented in the figure to compare the model predictions with experimental data. The results suggest that although Eqs.(19) and (20) are derived by assuming $|n - 1| \ll 1$ and $Re_n \ll 1$, they can be used safely in a range of $0.5 < n < 1$ and $10^{-3} < Re_n < 10^3$. In addition, these relationships are simpler than most of those given in the literature, and give appreciably good results also for Newtonian liquids within a range of acceptable error.

5. CONCLUSION

The main conclusions of above derivations may be summarized as follows:

1. Choosing a convenient velocity-profile- equation should be first step in the derivation of a drag coefficient statement for a spherical solid particle moving in non-Newtonian liquids. The resultant equations should be compared with experimental data and must be correlated as required.

2. The drag correction coefficient, X_n, may assumes values such that $X_n > 1$, or $X_n < 1$, or $X_n \approx 1$. The above-developed Eq (20) furnishes all these values depending on n and Re_n.

3. The drag coefficient statement, $C_D = f(n, Re_n)$, are also compatible with standard drag curve of Newtonian liquids in the range of $1 < Re_n < 10^3$. Therefore they may also be used in practical engineering applications of Newtonian liquids.

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