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Research Article

Analyzing of Usage Effect of the Distribution Functions for SMDO Algorithm via Benchmark Function with Matlab Toolbox

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ABSTRACT

This paper presents solution comparisons of benchmark functions by using stochastic multi-parameters divergence (SMDO) method with different distribution functions. Using benchmark functions is an important method in measuring the effectiveness of algorithms. Because benchmark functions are used by all algorithm producers while trying their algorithms and this provides a good tool for the others to compare their algorithms with similar procedures. Benchmark functions are used in this paper for the main purpose of analyzing randomization process. It is known that distribution functions take place a vital role in getting random numbers. These random numbers are used in stochastic methods through specifying step size. It is believed that a suitable random number acquisition process can support the search processes of algorithms. In this study the effects of distribution functions on benchmark functions are analyzed. For this purpose, a program is developed with MATLAB. The comparisons via the help of this program is shown in tabular form. The results are analyzed from the viewpoint of whether developing the randomization process makes contribution to problem solving power of algorithms. In this study SMDO algorithm is analyzed with different distribution functions by using different benchmark functions. In addition, in the study, a useful friend-friendly Matlab toolbox is proposed in which SMDO algorithm can be tested over different benchmark functions according to different distribution functions. (https://www.mathworks.com/matlabcentral/fileexchange/75044-smdo-with-distribution-function-for-benchmarking)

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Introduction

Engineering problems can be solved by stochastic or analytical methods depending on the nature of the problem. Analytical methods are used if the mathematical model and constraints of the related problem are fully known. [1], [2]. However, if the problem model and its constraints are not known, numeric methods are used. [3], [4]. In addition, in some cases, it is necessary to use semi-analytical and semi-numerical methods, which include both analytical and numerical structures. [5], [6].

However, the real world engineering problems generally cannot be solved with pure analytic methods. Because sometimes the known parameters are inadequate or there can be distortions which change the dynamics of the problem. Numerical methods search the solution space of the problem with a disciplined way.

Numerical optimization algorithms can actually be examined in two groups. These are stochastic and deterministic methods. There are many subtitles in the related main headings. In general, stochastic and metaheuristic methods are preferred for solving engineering problems in the literature. Because algorithms in this structure can easily solve the uncertainties in engineering problems. Fire fly [7], flower pollination [8], forest optimization [9] can be shown as example for the stochastic optimization algorithms. Tabu search [10], [11] bat inspired [12], harmony search [13] can be shown as example for the metaheuristic optimization algorithms. Especially in recent years, an extremely important increase has been in the use of algorithms inspired by nature. [14], [15].

However, instead of recommending new nature-inspired algorithms, analytical approaches must be found to mathematically improve algorithms which are inspired from nature. Because there can be an infinite approximate method can have inspired from our living environment world. Therefore, in addition to proposing new algorithms, structures that will improve the performance of existing numerical, stochastic optimization algorithms should be proposed. Thus, the SMDO method, which is one of the numerical methods, is in our focus to analyze in this paper with using different distribution functions. It is a powerful algorithm whose adequacy is proven in real engineering problems.

As it is known, SMDO algorithm makes random movements according to uniform distribution while scanning parameter vector spaces. The uniform distribution performs movements by using relative random values between [0-1]. At this point, there are many distribution functions in the literature. These distribution functions, which are analytic approaches with different mathematical backgrounds, can derive values such as uniform distribution. In this paper the SMDO method is selected to analyze this process. In the numerical optimization algorithms, it is well known that the random steps take a very important place in the process. But this randomization generally is made by uniform distribution. By analyzing this aspect of the algorithms, it is hoped that it can be opened new doors for us to strengthen our algorithms. It is believed that this issue can be realized by using appropriate distribution function in the algorithm. In this paper different distribution functions are analyzed with SMDO algorithm to make a clear perspective from this view. It is hoped that by specifying a relation between the algorithm and the distribution function will help in the process of making more analytical and strong random steps. In addition, a user-friendly Matlab toolbox where SMDO algorithm can be tested over different benchmark problems according to different distribution functions has been proposed in the study.

The rest of the paper is organized as follows; Section 2 presents the benchmark functions that we will use in this analysis. In Section 3; SMDO method is briefly explained and besides this its usage details are explained. In section 4, the solutions of different methods with different benchmark functions by using uniform distribution function are compared. In section 5, SMDO method is tried with different distribution functions by the help of different benchmark functions. In Section 6, SMDO Toolbox Program is introduced by the help of which SMDO can be tried with different benchmark functions and with different distribution functions. And finally in Section 7 includes the conclusion.

The Characteristics of Distribution Functions

As it is stated before, in this study the randomization process is aimed to be analyzed. At this point the random number acquisition process of the algorithms takes an important place. Generally, the random numbers are acquired through uniform distribution in stochastic algorithms. It is known that the characteristics of the random number acquisition process is specified by the used distribution function. To analyze this effect, four different distribution function is tried on SMDO algorithm. These are normal distribution, beta distribution, binomial distribution and extreme value distribution. The characteristics of these distribution functions are described below [16].

The normal distribution function is also called the Gaussian distribution function and it is a two-parameter family of curves. Normal distribution function is used for central limit theorem.[17]. Mathematical formulation of normal distribution probability density function (pdf) is given as follows:

$$y = f(x|\mu,\sigma) = \left(\frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}\right), \text{ for } x \in R \qquad (1)$$

where 'μ' stands for mean of the distribution; 'σ' stands for standard deviation of the distribution. The standard normal distribution function has zero mean and unit standard deviation.

Beta distribution function is a family of non-zero curves defined between 0 and 1. Mathematical formulation of beta distribution probability density function (pdf) is given as follows:

$$y=f(x|a,b) = \frac{1}{B(a,b)} x^{a-1}(1-x)^{b-1} I_{[0,1]}(x) \qquad (2)$$

where 'a' is the first shape parameter; 'b' is the second shape parameter. These parameters only affect the shape of the distribution and they do not have the effect of shifting or the stretching the distribution. In this formula $B(\cdot)$ is the Beta function and $I_{[0,1]}(x)$ stands for indicator function. This indicator function provides the nonzero probability between the range (0,1). [18].

Binomial distribution is the generalized version of Bernoulli distribution and it is a two-parameter family of curves. [19]. Mathematical formulation of binomial distribution probability density function (pdf) is given as follows:

$$y=f(x|N,p) = \binom{N}{x} p^x(1-p)^{N-x} \; ; x=0,1,2,..,N \qquad (3)$$

where 'x' is the number of successes in 'N' trials of a Bernoulli process with probability of success 'p'. So 'N' stands for number of trials and 'p' stands for the probability of success in a single trial.

The extreme value distribution is generally used to model the largest and smallest value in randomly distributed sets. [20]. Mathematical formulation of extreme value distribution probability density function (pdf) is given as follows:

$$y=f(x|\mu,\sigma)=\sigma^{-1}\exp\left(\frac{x-\mu}{\sigma}\right)\exp\left(-\exp\left(\frac{x-\mu}{\sigma}\right)\right) \qquad (4)$$

In this formula 'μ' stands for the location parameter and 'σ' stands for the scale parameter. This form of probability density function can be used for modeling the minimum value. Negative of the original values can be used for modeling the maximum value.

Review of the SMDO Method and Usage of Distribution Function

SMDO algorithm is a stochastic optimization algorithm that works with the set and trial mechanism [21], [22]. In Figure 1 the pseudo code of the SMDO algorithm is given to visualize the algorithm.

The SMDO algorithm was first used in the integer-order controller design relative to the fractional-order reference model [21]. The SMDO algorithm was designed for fractional-order PID simulation models using a two-stage master slave approach. [22]. Later, SMDO algorithm was developed and fractional order controller design was realized for main and yaw rotors of TRMS system. [23]. In order to increase control performance by changing control structures in the system, many control structures with two degrees of freedom have been firstly fractional ordered and then fractional-order controllers are designed with

SMDO algorithm to these structures [24], [25]. A fractional order model structure was created by using the simple circuit model approach for the receptor ligands. [26]. In fact, a fractional order PID controller is designed for a basic simulation model to see the effect of some different distribution functions with the SMDO algorithm [27]. In this study, many different distribution functions were used for many different benchmark functions.

```
Start
 Parameter Initialization
 Distribution Function Selection
 Starting Point
  if E(X^n) > E_Min
  Adaptation; Go to Adapted
      v= v+1;
      if v>parameterCount
        v=0;
      end
      Create the Step Size with Selected Distribution Function
      Compose ΔX^n
      Forward test
      Create the step size according to selected distribution function
      if E(X^n +ΔX^n) - E(X^n)<0
        X^(n+1) = X^n + ΔX^n
        Go to Starting Point
      end
      Backward test:
      Create the step size according to selected distribution function
      if E(X^n +ΔX^n) - E(X^n)<0
        X^(n+1) = X^n - ΔX^n
        Go to Starting Point
      end
   Go to Starting Point
   Adopted
   end
End
```

Fig. 1. Pseudo code of SMDO Algorithm

Comparisons SMDO Algorithm Performance According to Different Distribution Functions

In this study, SMDO algorithm has been updated with ‘Normal Distribution’, ‘Beta Distribution’, ‘Binomial Distribution’ and ‘Extreme Value Distribution’ functions. The uniform distribution structure working in the system has been replaced with these distributions. Statistical analyzes were carried out to establish consistency of results during operation. The proposed SMDO algorithm has been run several times and the best value has been recorded. Using these best error values matrix, 1st statistical moment value (mean), 2nd statistical moment value (standard deviation), 3rd statistical moment (skewness) and 4th statistical moment (kurtosis) values were calculated. In fact, when making statistical analyzes, 1st and 2nd statistical moments are generally used. In this study, the 3rd and 4th statistical moments are used in order to further understand the results and to make more analysis of the distribution function used. As it is known, skewness shows symmetry and kurtosis shows sharpness.

First of all, the SMDO algorithm is well equipped to find much better values for related benchmark functions with much iteration. However, in this study, multiple analyzes were made by taking iteration numbers less. In this study it is not intended to compare the SMDO algorithm with other algorithms in the literature. The aim of the study is to analyze the contribution of different distribution functions to the performance of the SMDO algorithm.

In Table 1, there is the SMDO algorithm executed with normal distribution. SMDO algorithm was run for 20 different benchmark functions 20 times according to uniform distribution and the average value of the results was calculated and shown in the column named with ‘Classical SMDO with Uniform (Mean)’. Then, as shown in Table 1, the SMDO algorithm was run according to normal distribution and the resulting repetitive results were compared equally. For example, for Table 1, the Ackley benchmark function is resolved with the original SMDO algorithm and the obtained value is given in the 1st row of the table under ‘Classical SMDO with Uniform (Mean)’ column. Its value is 0.0079. Then, the SMDO algorithm was updated with normal distribution, the Ackley function was resolved with this updated form and the value of 0.0112 was obtained and it is given in the first column in the table. The classical SMDO algorithm has derived better results for the Ackley function. In addition, with the values obtained as a result of 20 times repeated tests 1st, 2nd, 3rd and 4th statistical moment values are given for Ackley function. The algorithm was then run for the Beale benchmark function in a similar way, and firstly it is achieved 0.0065 mean value with the Classical SMDO algorithm. Then, the SMDO algorithm which is updated with the normal distribution was run and as a result, it was obtained as 0.0045 value as given in the second row in Table 1. Similarly, as a result of 20 iterations, 2nd, 3rd and 4th statistical moments are obtained. As shown in the table, painted in green, normal distribution function based SMDO algorithm produced better results in 13 of 20 benchmark functions than classical SMDO, and in other 7 benchmark functions where it did not produce better results, it produced results very close to classical SMDO. So it can be seen from the results, the effect of different distribution functions on optimization algorithms is clearly seen. In order to make the results more reliable, the algorithm has been run many times under the same conditions and the results have been obtained accordingly. As it can be seen from the results, the use of different distribution functions is in a direction that favors the algorithm performance as a result of statistical analysis. For example, by using normal distribution instead of uniform distribution for 20 different benchmark functions, in 13 benchmark functions in error averages there seems to be improvement. As can be seen in the table, the variance values of the related solutions are also lower than the first case.

In Table 2 below, there is the SMDO algorithm executed with beta distribution. Our reference point in this table is the Classical SMDO result which is given in each row in the column named with ‘Classical SMDO with Uniform (Mean)’. As in Table 1, the algorithm was run in a similar way, and in 15 of 20 benchmark functions, beta results were found with better results than conventional SMDO. This is another indication of the effect of the distribution function.

In Table 3 below, there is the SMDO algorithm executed with binomial distribution. Our reference point in this table is the Classical SMDO result which is given in each row in the column named with ‘Classical SMDO with Uniform (Mean)’. As in Table 1 and Table 2, the algorithm was run in a similar way, and in 4 of 20 benchmark functions, binomial results were found with better results than conventional SMDO.

In Table 4 below, there is the SMDO algorithm executed with extreme value distribution. Our reference point in this table is the Classical SMDO result which is given in each row in the column named with ‘Classical SMDO with Uniform (Mean)’. As in Table 1, Table 2 and Table 3 the algorithm was run in a similar way, and in 12 of 20 benchmark functions, extreme value results were found with better results than conventional SMDO. So this results can be evidence of another indication of the effect of the distribution function.

As can be seen from the tables, the variance is actually large in the values obtained with the uniform distribution function, which shows that the data set, that is, the values used during the optimization, are scattered. In stochastic i.e. numeric optimization algorithms, the main reason for making the statistical analyses for all results is whether the results will be stable when running the algorithm again or using it for other problems.

Therefore, in this study, the skewness and kurtosis results, which are the 3rd and 4th statistical moments, have been added to the mean and variances or standard deviation values. While examining symmetricity with skewness, kurtosis examines measure of peakedness of a probability distribution. In fact, these two values are not preferred in statistical analysis. However, in this study and many other studies, standard deviation and mean values are taken as statistical moment. Parameters and error values vary, especially when scanning the parameter vector space. This makes a difference according to the dynamics of the problem solved. Sometimes algorithms that do well in benchmarking trials may not perform very well in engineering problems. Therefore, it is thought that the performance analysis of the algorithm proposed or used for real-time engineering applications will be better by looking at the skewness and kurtosis values of the obtained parameters.

SMDO Toolbox

In this study, a toolbox was created for the SMDO algorithm, which operates according to different distribution functions. In this study, only some results are presented. However, in the presented toolbox, a toolbox has been created in which the algorithm can be run repeatedly according to 21 different distribution functions. The created toolbox (www.mathworks.com) is published on the site. The created toolbox is used as follows. The "Enter parameter count" section indicates how many times the algorithm will be run consecutively. The "Enter iteration number" section determines how many iterations a single cycle of the algorithm will be. The "Enter error limit" section is a special definition for the SMDO algorithm.

Table 1: Results of SMDO Algorithm with Different Benchmark Functions Using Normal Distribution Function

SMDO with Normal Distribution	***Global Min Value***	***Mean(First Moment)***	***Classical SMDO With Uniform (Mean)***	***Variance(Second Moment)***	***Skewness(Third Moment)***	***Kurtosis(Fourth Moment)***
SMDO with Ackley	0	0.0112	0.0079	0.01241	2.3824	10.2447
SMDO with Beale	0	0.0045	0.0065	0.0107	7.4792	74.6493
SMDO with Bohachecsky	0	0.0020	0.0018	0.0036	2.8920	13.0674
SMDO with Booth	0	0.0003	0.0010	0.0006	4.4293	29.6066
SMDO with Branin	0,3978	0.3910	0.4092	0.0554	-6.9216	48.9486
SMDO with DixonPrice	0	0.0039	0.0351	0.0053	3.3031	18.5576
SMDO with Easom	-1	-8.3330e-08	-2.9777e-08	1.16062e-07	-1.8169	5.3094
SMDO with GoldsteinPrice	3	3.2406	4.0397	0.6616	-0.9759	15.8988
SMDO with Griewank	0	3.1166e-05	2.8866e-05	2.8292e-05	0.6642	2.1408
SMDO with Hump	0	0.00253	0.0220	0.0027	2.3122	10.0579
SMDO with Levy	0	0.0002	0.0038	0.0004	5.1653	36.7470
SMDO with Matyas	0	3.24984e-05	3.3132e-05	3.0056e-05	0.6723	2.1384
SMDO with Perm	0	0.00756	0.0279	0.0093	2.8875	14.8627
SMDO with Powell	0	0.0005	0.0003	0.0013	4.7732	27.9739
SMDO with Rastrigin	0	0.0141	0.0067	0.02999	4.5009	30.5890
SMDO with Rosenbrock	0	0.1392	0.3961	0.1265	1.6300	7.3854
SMDO with Schwefel	0	813.7992	813.7996	115.3717	-6.9296	49.0199
SMDO with Shubert	-186,73	-22.31463	-18.3900	18.4898	-1.1027	2.8943
SMDO with Sphere	0	3.8099e-05	3.6085e-05	3.8897e-05	2.8275	20.0084
SMDO with Zakharov	0	4.5726e-05	4.0279e-05	6.4747e-05	4.3740	31.2018

Table 2: Results of SMDO Algorithm with Different Benchmark Functions Using Beta Distribution Function

SMDO with Beta Distribution	***Global Min. Val.***	***Mean(First Moment)***	***Classical SMDO With Uniform (Mean)***	***Variance(Second Moment)***	***Skewness(Third Moment)***	***Kurtosis(Fourth Moment)***
SMDO with Ackley	0	0.0048	0.0079	0.00479	1.5709	5.3299
SMDO with Beale	0	0.0011	0.0065	0.003025	10.2732	128.1170
SMDO with Bohachecsky	0	0.0018	0.0018	0.002896	3.06244	14.8792
SMDO with Booth	0	9.4914e-05	0.0010	0.00013	3.8692	22.8552
SMDO with Branin	0,3978	0.3908	0.4092	0.05541	-6.9237	48.9667
SMDO with DixonPrice	0	0.00099	0.0351	0.001649	3.5398	17.58250
SMDO with Easom	-1	-1.4773e-08	-2.9777e-08	1.0938e-08	-0.8225	2.6445191
SMDO with GoldsteinPrice	3	3.0417	4.0397	0.4760	-4.7354	34.1134
SMDO with Griewank	0	3.3553e-05	2.8866e-05	3.1356e-05	0.5283	1.8592
SMDO with Hump	0	0.00186	0.0220	0.0032	3.1804	13.8229
SMDO with Levy	0	0.0001	0.0038	0.0002	4.35260	24.2882
SMDO with Matyas	0	3.555e-05	3.3132e-05	3.0747e-05	0.5315	1.9206
SMDO with Perm	0	0.00311	0.0279	0.0070	8.2227	86.2100
SMDO with Powell	0	0.0002	0.0003	0.0011	12.2108	163.6566
SMDO with Rastrigin	0	0.00128	0.0067	0.00345	6.15650	46.1952
SMDO with Rosenbrock	0	0.3233	0.3961	0.1038	-0.6704	4.05219
SMDO with Schwefel	0	813.7992	813.7996	115.3717	-6.9296	49.0199
SMDO with Shubert	-186,73	-11.54244	-18.3900	5.0931	0.6507	2.21161
SMDO with Sphere	0	3.3593	3.6085e-05	3.2951e-05	1.3627	7.1360
SMDO with Zakharov	0	2.7623e-05	4.0279e-05	2.9612e-05	2.2103	13.0656

Table 3: Results of SMDO Algorithm with Different Benchmark Functions Using Binomial Distribution Function

SMDO with Binomial Distribution	***Global Min. Val.***	***Mean(First Moment)***	***Classical SMDO With Uniform (Mean)***	***Variance(Second Moment)***	***Skewness(Third Moment)***	***Kurtosis(Fourth Moment)***
SMDO with Ackley	0	0.1056	0.0079	0.5248	4.7737	23.8915
SMDO with Beale	0	5.942	0.0065	7.0001	0.3370	1.11486
SMDO with Bohachecsky	0	0.02941	0.0018	0.1538	6.7805	58.9932
SMDO with Booth	0	0.1053	0.0010	0.3637	3.9940	19.5653
SMDO with Branin	0,3978	1.1269	0.4092	1.8028	4.5147	26.0063
SMDO with DixonPrice	0	0.4803	0.0351	0.2918	0.8972	4.72660
SMDO with Easom	-1	-2.1170e-08	-2.9777e-08	4.4813e-08	-4.34627	24.4713
SMDO with GoldsteinPrice	3	248.7368	4.0397	2854.2248	14.16455	201.75833
SMDO with Griewank	0	0.01319	2.8866e-05	0.0220	1.7918	6.0395

SMDO with Hump	0	0.2901	0.0220	0.1082	5.2853	38.8661
SMDO with Levy	0	0.1645	0.0038	0.3102	2.9967	15.5586
SMDO with Matyas	0	0.0019	0.0279	0.0110	5.5704	32.0303
SMDO with Perm	0	1.0699	0.0003	1.9015	6.0539	50.1328
SMDO with Powell	0	0.7457	0.0067	4.2443	5.5668	32.0037
SMDO with Rastrigin	0	0.04411	0.3961	0.2058	4.4399	20.7128
SMDO with Rosenbrock	0	64.0465	813.7996	114.9534	1.9502	5.8908
SMDO with Schwefel	0	789.9691	-18.3900	116.8345	-6.0896	41.3975
SMDO with Shubert	-186,73	-14.6498	3.6085e-05	10.8293	-2.6302	10.7316
SMDO with Sphere	0	0.01348	4.0279e-05	0.08305	9.0685	100.8007
SMDO with Zakharov	0	0.01573	0.0079	0.1058	9.7663	113.6100

Table 4: Results of SMDO Algorithm with Different Benchmark Functions Using Extreme Value Distribution Function

SMDO with Extreme Value Distribution	***Global Min. Val.***	***Mean(First Moment)***	***Classical SMDO With Uniform (Mean)***	***Variance(Second Moment)***	***Skewness(Third Moment)***	***Kurtosis(Fourth Moment)***
SMDO with Ackley	0	0.0142	0.0079	0.01537	2.14544	9.2053
SMDO with Beale	0	0.0874	0.0065	0.9954	14.0801	200.12788
SMDO with Bohachecsky	0	0.0023	0.0018	0.0045	3.15584	13.8019
SMDO with Booth	0	0.0003	0.0010	0.00056	3.85149	26.6506
SMDO with Branin	0,3978	0.3915	0.4092	0.05555	-6.9114	48.8571
SMDO with DixonPrice	0	0.00376	0.0351	0.0045	2.5793	12.739
SMDO with Easom	-1	-7.8097e-08	-2.9777e-08	1.1694e-07	-2.0064	6.1050
SMDO with GoldsteinPrice	3	3.5185	4.0397	1.0694	2.41662	21.346
SMDO with Griewank	0	0.00017	2.8866e-05	0.00096	9.1354	90.4217
SMDO with Hump	0	0.003568	0.0220	0.00478	3.01209	14.6803
SMDO with Levy	0	0.0003	0.0038	0.000401	2.92719	15.7236
SMDO with Matyas	0	3.3554e-05	3.3132e-05	3.0966e-05	0.542472	1.93375
SMDO with Perm	0	0.0085	0.0279	0.01061	2.52858	11.58471
SMDO with Powell	0	0	0.0003	0	NaN	NaN
SMDO with Rastrigin	0	0.02889	0.0067	0.0856	6.8884	62.67449
SMDO with Rosenbrock	0	0.1388	0.3961	0.14363	2.27359	10.0489
SMDO with Schwefel	0	813.7992	813.7996	115.3717	-6.929	49.0199
SMDO with Shubert	-186,73	-22.0883	-18.3900	19.2324	-1.19970	2.9960

SMDO with Sphere	0	6.7565e-05	3.6085e-05	0.000192	11.5382	151.334
SMDO with Zakharov	0	0.0003	4.0279e-05	0.00123	7.0040	55.8365

It ensures that the algorithm remains within the stability limits for the parameters. The "Enter Divergence Vector" section is a specially defined function for SMDO as the scanning speed of the parameter vector spaces of the parameters optimized in the algorithm. The "Enter benchmark type" section provides the user with the option to select the desired benchmark function [28] with a variable for 27 different benchmark functions. The "Enter distribution type" section can actually be shown as a first for such algorithms. The benchmark function that is desired to be used is automatically selected in the toolbox and its effect in terms of algorithm can be easily monitored. In this study, a program called SMDO Toolbox is developed to implement SMDO algorithm. By the help of this program SMDO is applied on different benchmark test functions with different distribution functions and results are gained to compare within each other. The program is developed with MATLAB. In the program input parameters are taken through graphical user interfaces. The GUI of the program can be seen in Figure 3.

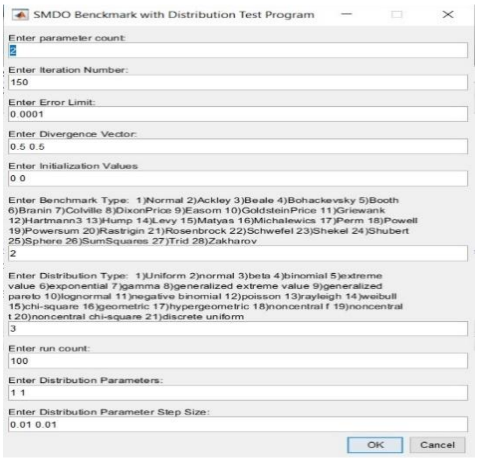

Fig. 3. GUI of SMDO Toolbox

Besides this, the mean, best and worst values are also shown with the help of this program to analyze and compare the results.

Conclusions

This paper presents the comparisons of solutions of different numerical optimization methods by using different benchmark functions with different distribution functions. SMDO algorithm has proven its success especially in a real world engineering problems such that controller design. This is a big deal because generally optimization problems are verified through benchmark test functions which are far away from solving real world problems efficiently. But if an algorithm does not contribute to a real world problem it remains restricted in the theoretical ground and we cannot use it in a practical way which is not preferred from an engineering perspective.

But the performance of SMDO like other numerical methods can be enhanced by developing its randomization step. In Section 4 it is shown that SMDO algorithm can work with most of the benchmark functions efficiently like other algorithms do. This comparison is made with uniform distribution function. From this table we can see that an algorithm which is eligible for real world problems can be applied to artificial benchmark functions. But to move this success one more step further, randomization process in the SMDO algorithm is focused on. For this SMDO algorithm is tried with different distribution functions by using different benchmark functions. When using different distribution functions a very important notion in statistics is analyzed which is statistical moments. The four statistical moments are shown in tabular form when using different distribution functions in Section 5. It is showed that the used distribution function in a numerical algorithm can change the solution. This effect can be positive or negative. It is seen that the distribution function which can be more suitable for an algorithm can be specified by analyzing the statistical moment of the data set produced. Because the statistical moments give the clues of algorithm's process characteristics. So by using the data produced in the algorithm makes us to analyze the characteristics of the randomization process of the algorithm.

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