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State variable distributed-parameter representation of transmission line for transient simulations

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Abstract

In this paper, a distributed-parameter state variable approach is used to calculate transients on transmission lines based on the concept of travelling waves. The method of characteristics for lossless line is used and the state equations are derived for the system. These equations are converted to a set of difference equations using the trapezoidal rule of integration and solved in time domain using LU decomposition. Single- and multi-phase transmission lines with various linear and non-linear terminations are considered in the illustrative examples. State-space modeling of transposed lines using modal decomposition is introduced and the effects caused by lumped parameter approximation of line losses are described. Comparisons with the results obtained by EMTP and conventional methods based on s-domain are given.

Key Words: *Transmission line modeling, state-space methods, power system transients.*

1. Introduction

Determination of accurate transient behavior of a transmission line requires precise representation of distributed parameters, as transmission lines are, by their nature, distributed parameter systems. A transmission line can be perfectly represented in frequency-domain with distributed parameters. However, it is it difficult to include the effects of nonlinear elements and switching conditions. Therefore, time-domain methods are preferred for transient simulation of transmission line systems.

For distributed parameter representation of a transmission line, the method of characteristics, which is based on wave propagation for a lossless line, forms the basis for time domain formulation. The method was first applied to hydraulic problems by Bergeron [1], and then used for transmission line transient simulations by Branin [2]. Later, Dommel proposed a nodal solution [3] by combining the method of characteristics for transmission lines and the trapezoidal rule of integration for lumped parameters. Based on this theory, Electromagnetic Transients Program (EMTP) has been developed [4], and is capable of simulating lossless or distortionless lines with distributed parameters.

Alternatively, the state-space techniques can be used for transient simulations of electrical circuits in time domain. Lumped RLC elements can be represented in the state-space methods accurately, but instead of distributed parameters, generally lumped parameter transmission line models constructed by cascaded connection of T-, π- or L-sections are used for transient analysis [5–8]. The accuracy of lumped parameter approximation depends on the line length and highest transient frequency component involved, and therefore, representation of transmission lines may require several line sections to achieve sufficient accuracy. On the other hand, numerous state variable approaches have also been proposed for representation of the transmission line by distributed parameters [9–11], but these methods are difficult and not practical to implement.

In this work, discrete-time state-space model of distributed parameter transmission line [12] is used for transient simulations of transmission lines. The method of characteristics is used in line modeling and the state equations are derived. These equations are discretized using the trapezoidal rule of integration and solved in time-domain using LU factorization. Several examples which involve single- and multi-phase transmission lines with different kind of linear and nonlinear terminations are given. The state-space analysis of transposed lines using modal transformations is described. The study also investigates the effects caused by lumped parameter approximation of line losses using s-domain analysis. Results indicate that the state variable approach can be conveniently applied to the transient simulation of transmission lines.

This paper is organized as follows. Following this introductory section, in the second section, a general description of distributed parameter transmission line is given and the method of characteristics for its analysis is described. In Section III, state-space representation of linear systems involving transmission lines is introduced. Solution of state equations by discretization using trapezoidal rule of integration is examined as a subject of Section IV. In Section V, the examples worked out by using the state-space techniques are presented and remarks on the performance of the state-space modeling of distributed parameter transmission line are described.

2. Distributed-parameter line model

The parameters of a uniform transmission line r, l, g and c (resistance, inductance, conductance and capacitance per unit length, respectively) are distributed perfectly along the line. The voltage and current at any point on the transmission line can be described by both space and time dependent partial differential equations (known as Telegrapher's Equations) and are written

$$-\frac{\partial v}{\partial x} = ri + l\frac{\partial i}{\partial t} \quad , \tag{1a}$$

$$-\frac{\partial i}{\partial x} = gv + c\frac{\partial v}{\partial t}. \tag{1b}$$

The method described by Bergeron applies to lossless lines ($r = 0,\ g = 0$) for which (1a) and (1b) yield the forward and backward traveling waves as [1,3]

$$v(x,t) + Z_0 i(x,t) = 2Z_0 f_1(x - \upsilon t), \tag{2a}$$

$$v(x,t) - Z_0 i(x,t) = -2Z_0 f_2(x + \upsilon t). \tag{2b}$$

Z_0 in above equations is the characteristic impedance and v is the phase velocity, which are defined in terms of per unit length parameters as

$$Z_0 = \sqrt{L'/C'}, \tag{3}$$

$$v = 1/\sqrt{L'C'}. \tag{4}$$

This approach forms the basis for the method of characteristics and this formulation can be expressed as an equivalent network for the line between its sending and receiving ends.

Consider a transmission line between node k and m with travel time $\tau = \ell/v = \ell\sqrt{lc}$, where ℓ is the line length. The line can be modeled by an equivalent network composed of impedance elements and current sources [3], which describes the lossless line at its terminals. The equivalent two-port network model for a lossless line is illustrated in Figure 1. Current sources I_k and I_m at instant t in this network model are determined from the past history at time $(t-\tau)$ via the system

$$I_k(t-\tau) = -(1/Z_0)v_m(t-\tau) - i_{m,k}(t-\tau), \tag{5a}$$

$$I_m(t-\tau) = -(1/Z_0)v_k(t-\tau) - i_{k,m}(t-\tau). \tag{5b}$$

The method of characteristics is applicable for simulation of a lossless or distortionless line. However, by treating a lossy line as segments of ideal lines connected with discrete resistors, the method of characteristics remains applicable for simulation of lossy lines. The effective series resistance of the line can be represented by lumped resistors, and the effect of line attenuation is approximated by adding half of the total line resistance $R' = r\ell$ at the ends of line, or more precisely by adding $R'/4$ at the terminals and $R'/2$ in the middle of the line. In this case, Z_0 in Figure 1 is replaced by the relation $Z = Z_0 + R'/4$ and, respectively, past history currents are determined as [4]

$$I_k(t-\tau) = \left(\frac{1+h}{2}\right)\left[(-1/Z)v_m(t-\tau) - hi_{m,k}(t-\tau)\right] + \left(\frac{1-h}{2}\right)\left[(-1/Z)v_k(t-\tau) - hi_{k,m}(t-\tau)\right], \tag{6a}$$

$$I_m(t-\tau) = \left(\frac{1+h}{2}\right)\left[(-1/Z)v_k(t-\tau) - hi_{k,m}(t-\tau)\right] + \left(\frac{1-h}{2}\right)\left[(-1/Z)v_m(t-\tau) - hi_{m,k}(t-\tau)\right], \tag{6b}$$

where $h = (Z_0 - \frac{r\ell}{4})/(Z_0 + \frac{r\ell}{4})$. Experience with this model indicates that the error incurred by lumping the series resistance is acceptable as long as $r\ell << Z_0$ [4].

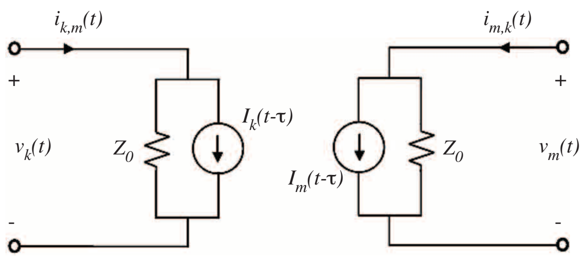


Figure 1. Equivalent two-port network for a lossless line.

3. State-space representation

The state equations for a circuit involving current and voltage sources and linear RLC elements are written as [13]

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t), \tag{7a}$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t), \tag{7b}$$

where $\mathbf{x}$ is the state vector, $\mathbf{y}$ is the vector of output variables, $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ are coefficient matrices with proper dimensions, and $\mathbf{u}$(t) represents the vector of inputs. The characteristic model of transmission line involves current sources and impedance elements only; hence it can be easy implemented. By replacing the transmission line with its equivalent impedance network, a circuit model involving current and voltage sources and RLC elements is obtained. Then, the state equations are derived by choosing inductor currents and capacitor voltages properly as state variables. Once state variables are determined, other currents and voltages in the circuit rather than the state variables can be calculated in terms of state variables and inputs using the output equations.

Compared to conventional systems, the main difference in state equations of systems involving a transmission line is the existence of past history currents, which are required to be updated at every time step of the solution. This requires discretization of the state equations, which can be achieved by performing integration of the state equations numerically. The trapezoidal rule of integration is one of the numerical integration methods that can be used for this purpose, and in this study this method has been chosen for its efficiency, accuracy and good numerical stability characteristics [14].

Let $\mathbf{x_k}$ at time ($t_k = k\Delta t$) be known. Using the trapezoidal rule of integration, (7a) can be easily transformed to a set of difference equations and the response of a system at $t_{k+1} = (k+1)\Delta t$ is determined from the following equation:

$$(\mathbf{I} - \frac{h}{2}\mathbf{A})\mathbf{x}_{k+1} = (\mathbf{I} + \frac{h}{2}\mathbf{A})\mathbf{x}_k + \frac{h}{2}\mathbf{B}(\mathbf{u}_k + \mathbf{u}_{k+1}), \tag{8}$$

where $h = \Delta t$ is the step length and subscript k and $k+1$ for the state vector $\mathbf{x}$ and the vector of inputs $\mathbf{u}$ denotes the values at t_k and t_{k+1}, respectively. Note that the value of past history currents at state t is known from past history at $t-\tau$. The state vector at discrete time points is repetitively calculated starting from a known state at k=0 to obtain a time domain response of the system. This requires simultaneous solution of the linear set of algebraic equations in (8), which is written for simplicity as

$$\bar{\mathbf{A}}\mathbf{x}_{k+1} = \mathbf{b}_k, \tag{9}$$

where,

$$\bar{\mathbf{A}} = (\mathbf{I} - \frac{h}{2}\mathbf{A}), \tag{10}$$

and

$$\mathbf{b}_k = (\mathbf{I} + \frac{h}{2}\mathbf{A})\mathbf{x}_k + \frac{h}{2}[\mathbf{B}(\mathbf{u}_k + \mathbf{u}_{k+1})]. \tag{11}$$

To obtain the response of the system, (9) is solved for the state vector $\mathbf{x}_{k+1}$ repetitively starting from k=0. Note that the matrix $\bar{\mathbf{A}}$ is constant for a linear system; it is evaluated before the simulations begin. The vector

$\mathbf{b}_k$ must be evaluated at each discrete point of time, and the past history currents of the characteristic model of the line need to be updated at every time step using (5) for lossless and (6) for lossy lines. If coefficient matrix $\mathbf{A}$ is diagonal, solution of (9) is straightforward. For a general matrix, triangular factorization such as LU decomposition may reduce the computational costs.

4. Applications and results

4.1. Representation of single-phase transmission line

In this example, distributed parameter state-space representation of a single-phase transmission line is described and the effect of lumped parameter representation for the line losses is investigated. A 300 km long single-phase power transmission line driven by a voltage source with a series inductance L and a parallel RC load, shown in Figure 2, is considered an illustrative example. The line parameters are chosen as $r = 0.02\ \Omega$/km, $l = 1.14$ mH/km and $c = 9.8$ nF/km. Replacing the transmission line with its characteristic model, the equivalent circuit shown in Figure 3 is obtained. Choosing inductor current and capacitor voltage as the state variables the state equations are then written as

$$\frac{d}{dt}\begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix} = \begin{bmatrix} -\frac{Z}{L} & 0 \\ 0 & \frac{-1}{C}\left(\frac{Z+R}{ZR}\right) \end{bmatrix} \begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{L} & \frac{Z}{L} & 0 \\ 0 & 0 & -\frac{1}{C} \end{bmatrix} \begin{bmatrix} v_S(t) \\ I_1(t-\tau) \\ I_2(t-\tau) \end{bmatrix}. \tag{12}$$

A step function is considered for the source voltage, and sending-end and the receiving-end currents and voltages of the transmission line are computed using a 5 μs step length. Line losses are not included in the first simulation. Note that the sending-end current is identical with state variable i_L and the receiving-end voltage is identical with v_C, hence they can be obtained from the solution of (17) directly. The sending-end voltage and the receiving-end current however, are determined in terms of the state variables and the vector of sources from the following output equations:

$$\begin{bmatrix} v_1(t) \\ i_{2,1}(t) \end{bmatrix} = \begin{bmatrix} Z & 0 \\ 0 & \frac{1}{Z} \end{bmatrix} \begin{bmatrix} i_L(t) \\ v_C(t) \end{bmatrix} + \begin{bmatrix} 0 & -Z & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_s(t) \\ I_1(t-\tau) \\ I_2(t-\tau) \end{bmatrix} \tag{13}$$

Figure 4a and b show the sending-end and receiving-end current and voltage waveforms of the line computed by the proposed approach. As it can be seen from the figures, the incident voltage and current waves arrive at receiving-end of the line at about $t = 1$ ms, which is approximately equal to the travel time of the line. Wave propagation continues until reaching a steady state, and as time increases, voltage and current at receiving-end become closer to expected values $v_S = 1$ V and $v_S/R = 1$ mA, which can be observed from the figures.

The receiving-end voltage of transmission line is also computed using some other solution methods for the state equations. The voltages computed by the trapezoidal rule of integration, forward Euler's method and Heun's method are shown in Figure 5; computer CPU times are 1.97, 1.92 and 1.95 s for trapezoidal rule of integration, Euler's method and Heun's Method, respectively. Although Euler method seems advantageous, it has been observed that the method fails when large step length (about 20 μs for this example) is used.

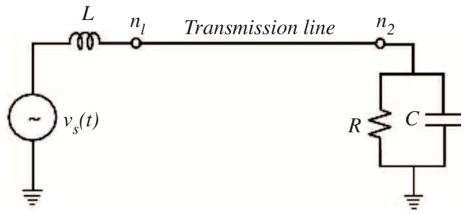


Figure 2. Single-phase line with terminations in Example 1. (L= 50 mH, R= 1 kΩ, C= 0.1 μF).

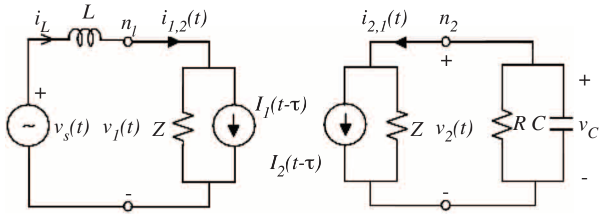
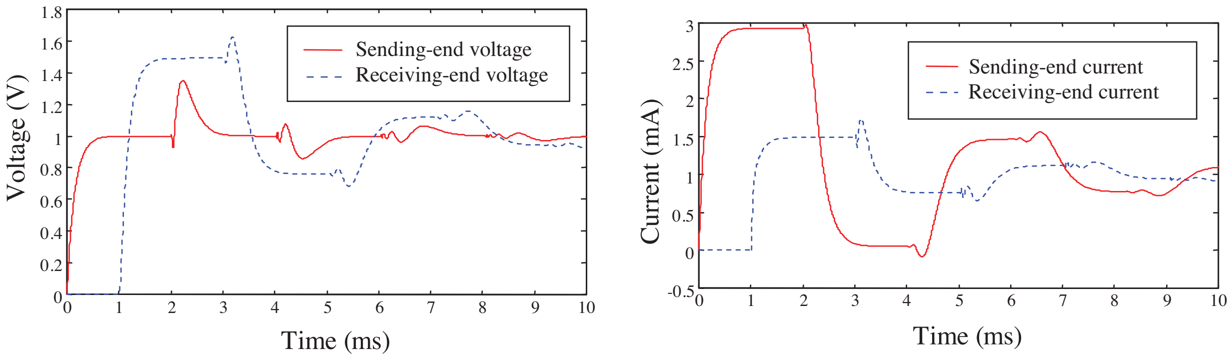


Figure 3. Equivalent circuit model of single-phase transmission system for state-space modeling.



(a) (b)

Figure 4. Sending-end and receiving-end voltage (a) and current (b) waveforms computed by state-space modeling.

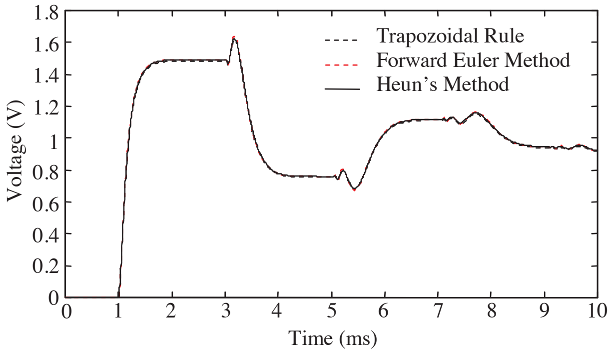


Figure 5. Receiving-end voltage waveforms computed by using different solution methods for the state equations.

The test circuit is considered for a second case to show how to investigate the effects of lumped parameter approximation of line losses. A 50 Hz input $v_s(t) = \cos \omega t$ is considered for this case. The accuracy of this approach is examined practically by comparing the simulation results with the waveform obtained by $s-$domain modelling (described in the Appendix) and using Fast Inverse Laplace Transform (FILT) for frequency- to time-domain conversions. Line losses are fully distributed in the case of s-domain simulations. Lumped elements are used in the case of state-space modeling; hence $Z = Z_0 + r\ell/4$ in (17) and past history currents are calculated from (9) and (10) accordingly. Figure 6 shows the line-end voltage computed by the state-space techniques and FILT. As it can be observed from the figure, the effect of lumped parameter representation of line losses is insignificant for this application.

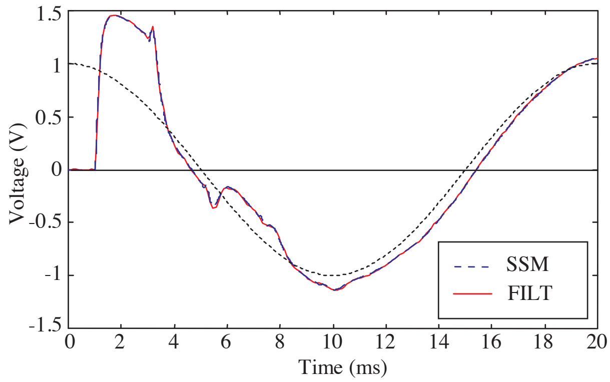


Figure 6. Receiving-end voltage waveforms of a transmission line with losses computed by state-space modeling (SSM) and s-domain simulations using FILT. (Dotted curve corresponds to the applied voltage wave.).

4.2. Representation of three-phase line

In this application, the state-space representation of a three-phase transmission line using modal transformation techniques is described. A 180 km long 3-phase, 50 Hz, 500 kV single circuit, transposed transmission line illustrated in Figure 7 is chosen for line energization studies, for illustration. It is assumed that the transmission line at no load is energized by a sinusoidal voltage source with series inductance $L_S = 34$ mH, when the voltage wave of phase-A is passing through its peak value. It possible to obtain the state-space response of the system in phase domain directly; however, there is mutual coupling between current and voltage of three phases, and because of this, analysis of such a system is difficult. The effect of coupling between the phases can be eliminated using phase to modal transformation techniques, which requires eigenvalue analysis [15]. By this way a three-phase transmission line can be decoupled into three independent modes and each mode is analyzed as a single phase line.

Let T_v and T_i be the modal transformation matrix for the voltage and current, respectively. The relationship between phase and modal voltages and currents can be described as

$$V_p = T_v V_m, \tag{14a}$$

and

$$I_p = T_i I_m, \tag{14b}$$

where subscript p and m denotes the phase and modal quantities, respectively. For a fully transposed three-phase transmission line modal transformation is not unique and the following transformation matrix

$T = T_v^{-1} = T_i$ may be used:

$$T = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & -1 & -1 \end{bmatrix}. \tag{15}$$

Using characteristic model for each mode of the line, modal circuits are constructed and the state equations are obtained. These equations are solved to obtain the state variables in the modal domain, and then desired output variables are determined. Modal to phase domain transformation is used to obtain the output voltages and currents in the phase domain.

The modal parameters for the overhead line considered in this example are:

$r^{(0)}$ =0.29 Ω/km, $r^{(1)}$ =0.0484 Ω/km,

$l^{(0)}$ =3.23 mH/km, $l^{(1)}$ =1.012 mH/km,

$c^{(0)}$ =7.66 nF/km, $c^{(1)}$ =11.86 nF/km,

where superscript (0) and (1) denote zero and positive sequence parameters, respectively. The state-space representation of the three phase system in this example is described in detail below.

Consider the equivalent circuit for the positive sequence network shown in Figure 8. The number of state variables for this circuit is one, and the state equation representing this circuit is obtained as

$$\frac{di_L^{(1)}(t)}{dt} = \frac{-Z^{(1)}}{L_s} i_L^{(1)}(t) + \frac{1}{L_s} v_S^{(1)}(t) + Z^{(1)} I_1^{(1)}(t - \tau^{(1)}), \tag{16}$$

where $Z^{(1)} = Z_0^{(1)} + r^{(1)}\ell/4$ and $\tau^{(1)} = \ell\sqrt{l^{(1)}c^{(1)}}$. Note that $L_s^{(0)} = L_s^{(1)} = L_s^{(2)} = L_s$. Once $i_L^{(1)}(t) = i_{1,2}^{(1)}(t)$ is determined, the past history current $I_2^{(1)}$ can be found using (6), and the receiving-end voltage in modal domain is obtained as

$$v_2^{(1)}(t) = -Z^{(1)} I_2^{(1)}(t - \tau^{(1)}), \tag{17}$$

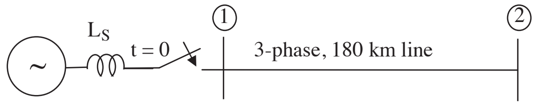


Figure 7. Three phase transmission system considered in Example 2.

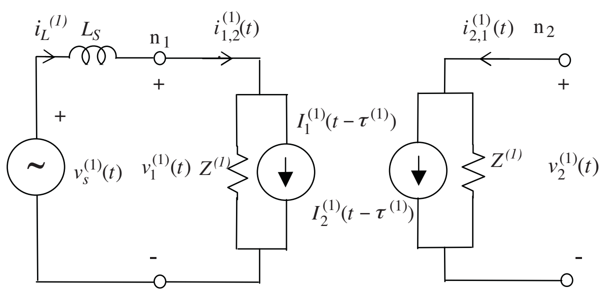


Figure 8. Equivalent circuit for the positive-sequence network of the system.

Similar expressions are derived for the other sequence networks, and as a consequence, the state equations representing the modal circuits may be written in matrix from as

$$\frac{d}{dt}\begin{bmatrix} i_L^{(0)}(t) \\ i_L^{(1)}(t) \\ i_L^{(2)}(t) \end{bmatrix} = \begin{bmatrix} \frac{-Z^{(0)}}{L_s} & 0 & 0 \\ 0 & \frac{-Z^{(1)}}{L_s} & 0 \\ 0 & 0 & \frac{-Z^{(2)}}{L_s} \end{bmatrix} \begin{bmatrix} i_L^{(0)}(t) \\ i_L^{(1)}(t) \\ i_L^{(2)}(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{L_s} & 0 & 0 & \frac{Z^{(0)}}{L_s} & 0 & 0 \\ 0 & \frac{1}{L_s} & 0 & 0 & \frac{Z^{(1)}}{L_s} & 0 \\ 0 & 0 & \frac{1}{L_s} & 0 & 0 & \frac{Z^{(2)}}{L_s} \end{bmatrix} \begin{bmatrix} v_s^{(0)}(t) \\ v_s^{(1)}(t) \\ v_s^{(2)}(t) \\ I_1^{(0)}(t-\tau^{(0)}) \\ I_1^{(1)}(t-\tau^{(1)}) \\ I_1^{(2)}(t-\tau^{(1)}) \end{bmatrix}, \tag{18}$$

and the output equations to obtain the receiving-end voltages in modal domain are written as

$$\begin{bmatrix} v_2^{(0)}(t) \\ v_2^{(1)}(t) \\ v_2^{(2)}(t) \end{bmatrix} = \begin{bmatrix} -Z^{(0)} & 0 & 0 \\ 0 & -Z^{(1)} & 0 \\ 0 & 0 & -Z^{(2)} \end{bmatrix} \begin{bmatrix} I_2^{(0)}(t-\tau^{(0)}) \\ I_2^{(1)}(t-\tau^{(1)}) \\ I_2^{(2)}(t-\tau^{(2)}) \end{bmatrix}, \tag{19}$$

where $Z^{(j)} = Z_0^{(j)} + r^{(j)}\ell/4$ and $\tau^{(j)} = \ell\sqrt{l^{(j)}c^{(j)}}$. After determination of $v_2(t)$ for each mode, modal-to-phase domain transformation is applied to obtain the response of the system in phase domain. A symmetrical source is considered in this example; therefore $v_s^{(0)}(t) = 0$. Receiving-end voltages computed by the state-space approach using step size Δt=10 μs are shown in Figure 9.

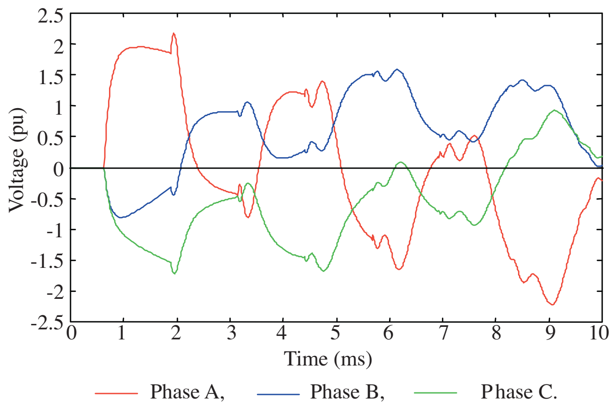


Figure 9. Line receiving-end voltages computed by state-space modeling.

4.3. Representation of nonlinear and time-varying elements

In this example, application of the state-space method for representation of nonlinear and time-varying elements is demonstrated. A nonlinear transmission system from [16], where the system involves a single-phase transmission line which is connected to a transformer at no load protected by 209 kV MOV surge arrester is considered. The transmission line is ℓ=2185.4 km in length and it is subjected to a 1.3/6.2μs double exponential voltage surge with an amplitude V=1560 kV. Line parameters are r=11.35 Ω/km, l=1.73 mH/km, and c=7.8 nF/km. The transformer is simply modeled by a capacitance to ground of C_T=6 nF. The test system is illustrated in Figure 10a, where R_a is used to represent the surge arrester and C_T represents the transformer capacitance shunt to the arrester. Replacing the transmission line with its characteristic model, the equivalent circuit for the system shown in Figure 10b is obtained. In this simulation line losses are taken into account hence Z in the equivalent circuit is taken to be $Z_0 + r\ell/4$. The nonlinear variation of the resistance representing the surge arrester is expressed as

$$R_a = 1.23 \times 10^{24} v^{-8.025}. \tag{20}$$

The state dimension for this system is one and the state variable is $v_C(t)$, which is equal to line receiving-end voltage. Single differential equation representing the state of the system is obtained as

$$\frac{dv_C(t)}{dt} = -\frac{R_a Z}{C_T(Z + R_a)} v_C(t) - \frac{1}{C_T} I_2(t - \tau), \tag{21}$$

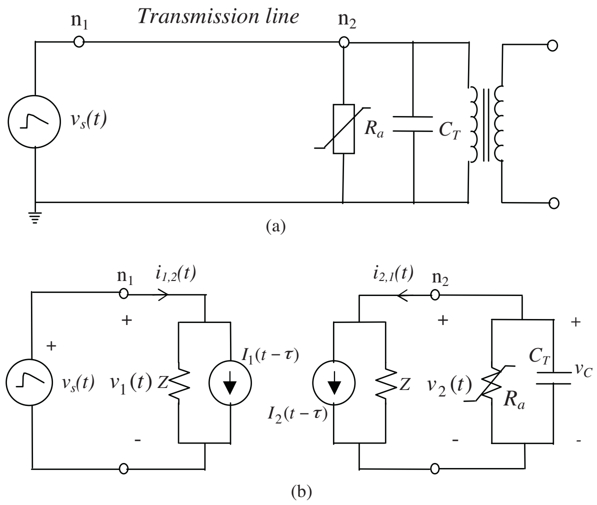


Figure 10. Single-phase transmission line with arrester considered in Example 3 and (b) its equivalent circuit.

Throughout the solution of this equation, $I_2(t - \tau)$ is calculated at each time step using (6) and the arrester voltage (voltage at line end) is obtained, which is equal to $v_C(t)$. The non-iterative state space approach used in [16] is used for the simulation of nonlinear element, where the same system was analyzed using lumped parameter approximation for the transmission line. In Figure 11, the receiving-end voltage obtained using state-space method with 0.05 μs step length and results produced using EMTP (with the same step length) are drawn on the same figure for comparison. EMTP results are obtained by using piecewise linear approximation of the arrester characteristics. As can be seen from the figure, except for some small deviations, a close agreement is observed between the results obtained by the two methods. The arrester voltage has been suppressed near 550 kV in both results, which is due to fast decrease in the nonlinear resistance representing the arrester above this voltage. Small differences are due to piecewise linear modeling of the surge arrester in the EMTP whereas assumed analytical variation of the surge arrester characteristics is treated exactly in the state-space simulations. Time-varying elements can be simulated in the same manner.

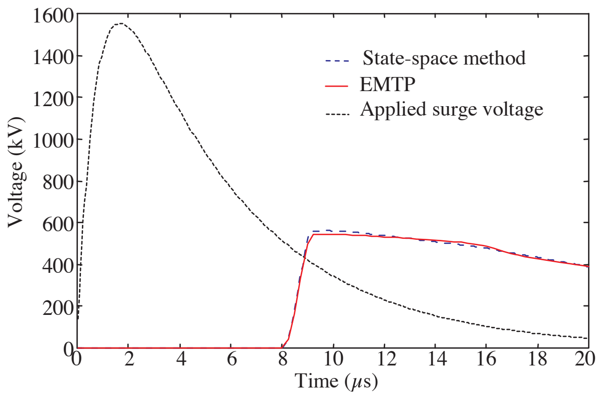


Figure 11. Effect of surge arrester on the surge voltage at line end.

5. Conclusions

A distributed-parameter state variable approach is presented for transient analysis of transmission lines in time domain. The method of characteristics for lossless line is used and the state equations are derived from the equivalent network. The trapezoidal integration is utilized to convert the state equations to a set of difference equations and then these equations are solved to obtain the response of the system. In the illustrative examples presented the state variable formulation of several circuits involving single- and multi-phase transmission lines with linear and nonlinear elements are described. As discretization of lumped RLC elements is not needed, the method has potential of becoming an alternative tool to the nodal solution method.

APPENDIX

A. s-domain transient simulations

Let us consider a transmission line with s-domain series impedance $z = r + sl$ and shunt admittance $y = g + sc$ per unit length. The equations for the terminal voltages and currents of the transmission line obtained from the frequency domain solution of (1a) and (1b) are written as

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} \tag{22}$$

where subscripts R and S stand for the receiving and the sending ends, respectively, and ABCD parameters in terms of hyperbolic functions are expressed as

$$A = D = \cosh \gamma d, \quad C = Z_0 \sinh \gamma d, \text{ and } D = Z_0^{-1} \cosh \gamma d,$$

where $Z_0 = \sqrt{Z/Y}$ is the characteristic impedance and $\gamma = \sqrt{ZY}$ is the propagation constant of the line.

The receiving-end voltage of the line shown in Figure 3 in s-domain is determined as

$$V_R = \frac{U(s)}{A + Z_S C + (1/Z_L)(B + Z_S D)} \quad , \tag{23}$$

where $U(s)$ is the source voltage in s-domain, $Z_S = sL$ is the source impedance and $Z_L = R/(1 + sCR)$ is the load impedance. Frequency to time domain conversion of s-domain expressions in (23) can be achieved by using the numerical method developed by Hosono [17], which is known as the Fast Inverse Laplace Transform (FILT).

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